Proof Complexity of Symbolic QBF Reasoning

Stefan Mengel and Friedrich Slivovsky

Symbolic Quantifier Elimination

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as in: *symbolic model checking*

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p	q
1	0
0	1
1	1

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$p \lor q$

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⇨ $p \lor q$ ⇨

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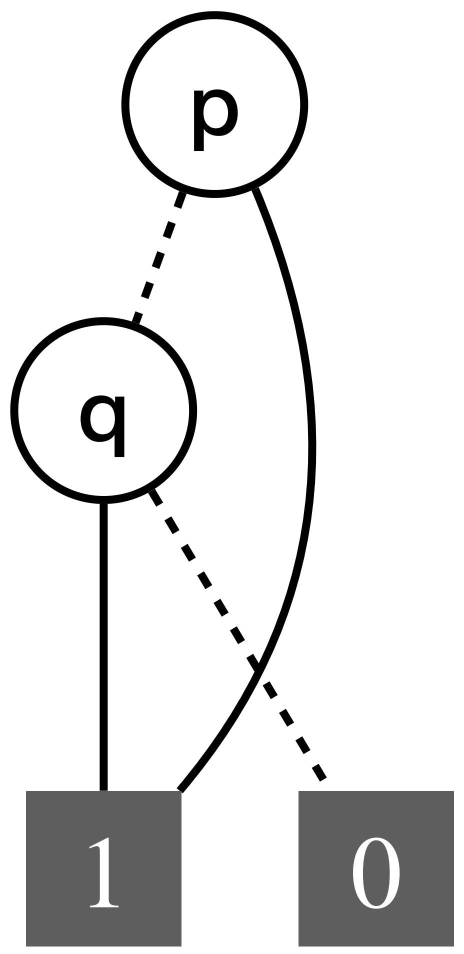

Symbolic Quantifier Elimination

QBDD Pan & Vardi 2004

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$$\forall x \exists y \,.\, (x \vee \neg y) \wedge (\neg x \vee y)$$

Symbolic Quantifier Elimination

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$\forall x \exists y \,.\, (x \lor \neg y) \land (\neg x \lor y)$

$\forall x \,.\, \varphi[y \leftarrow 0] \lor \varphi[y \leftarrow 1]$

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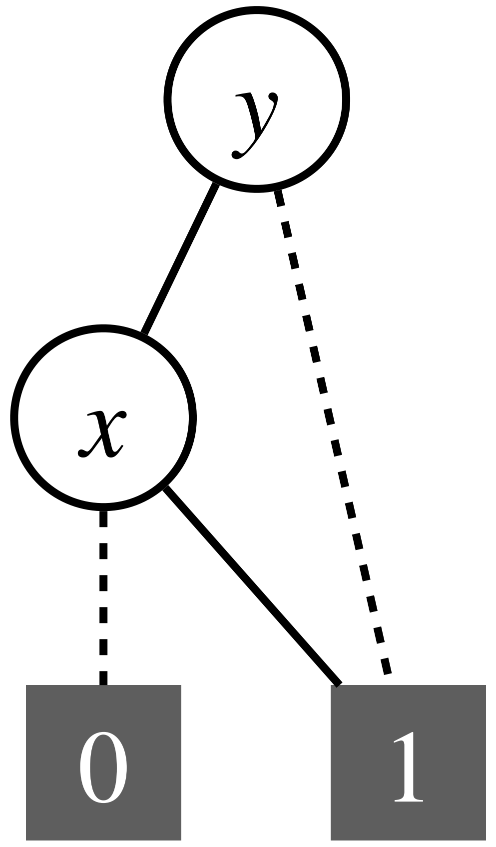

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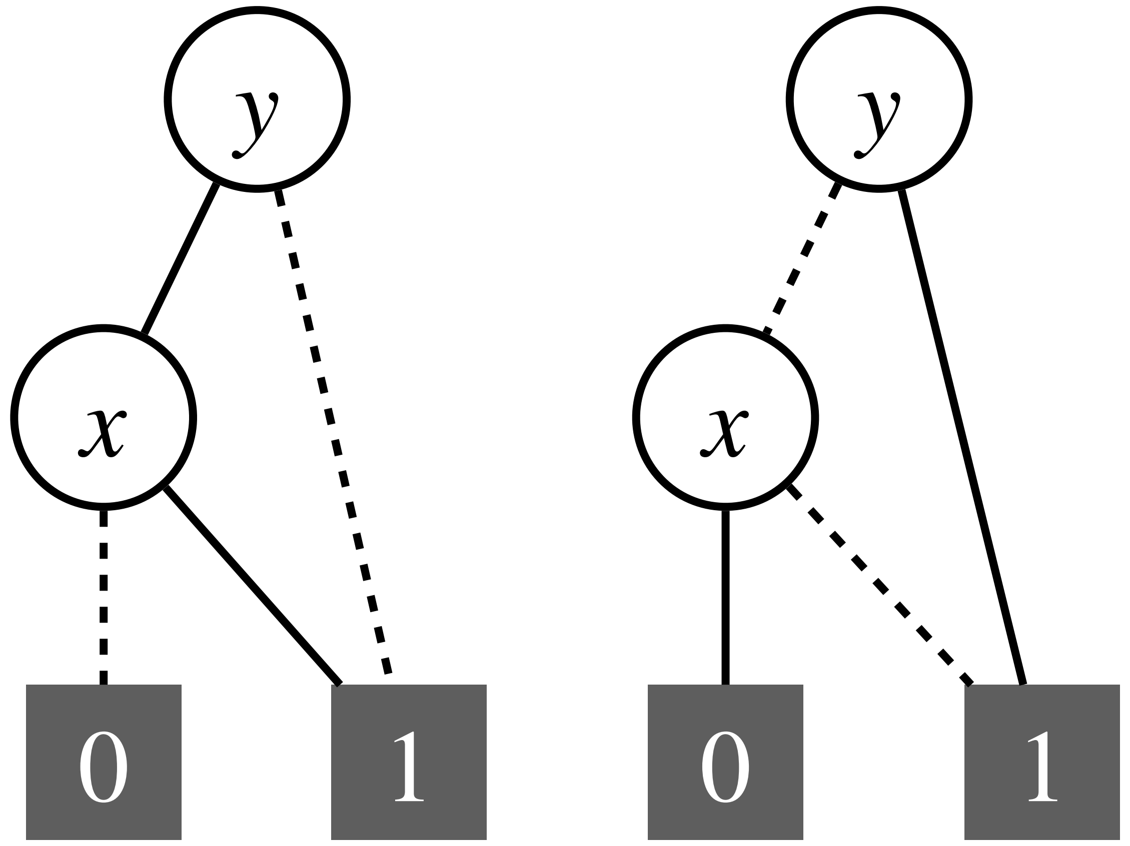

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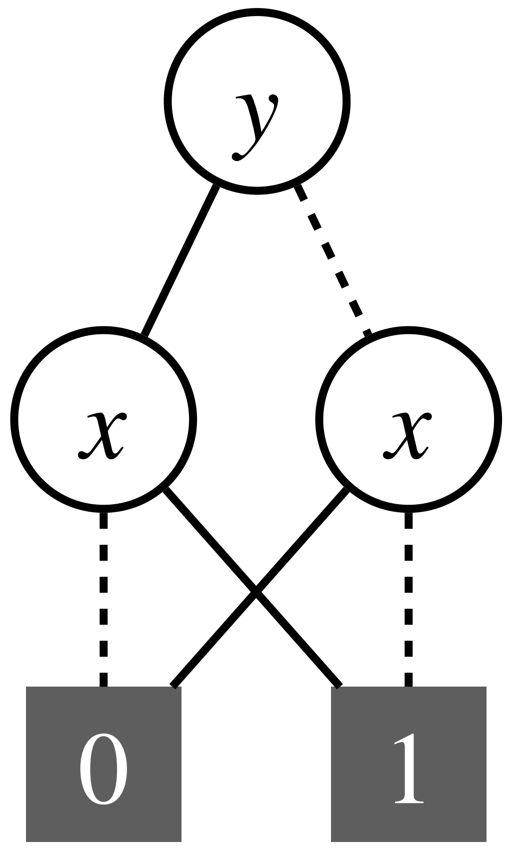

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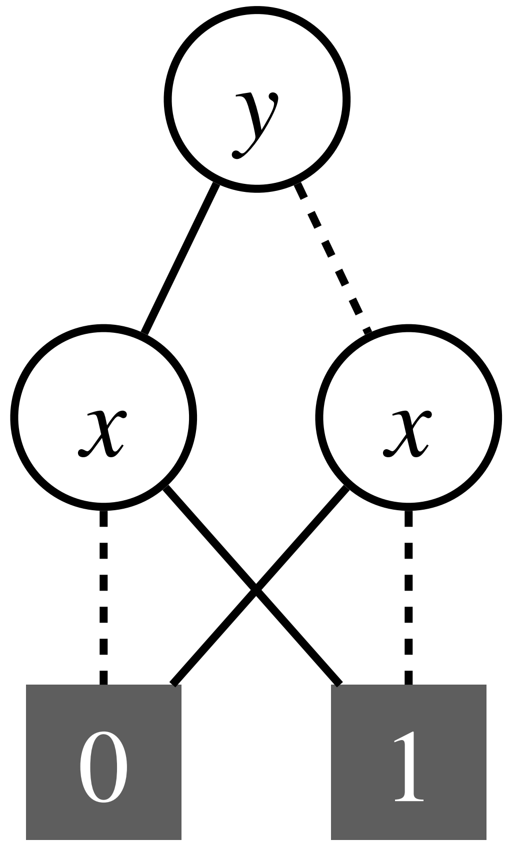

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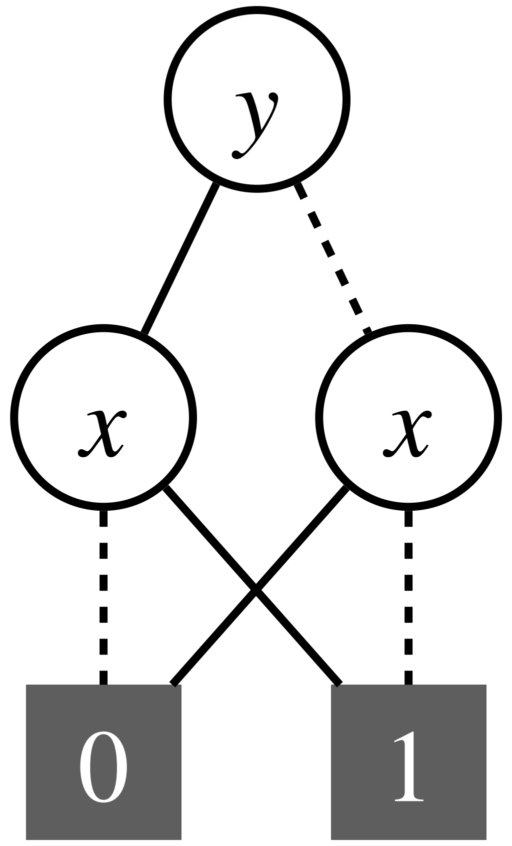

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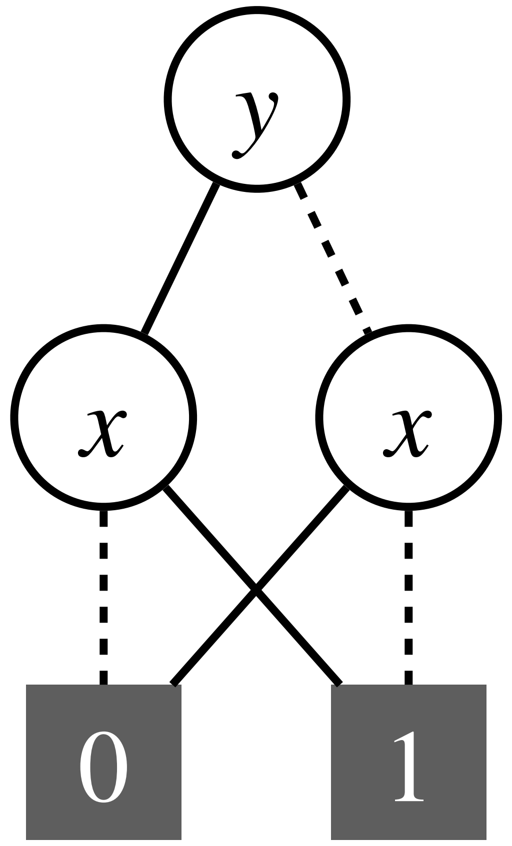

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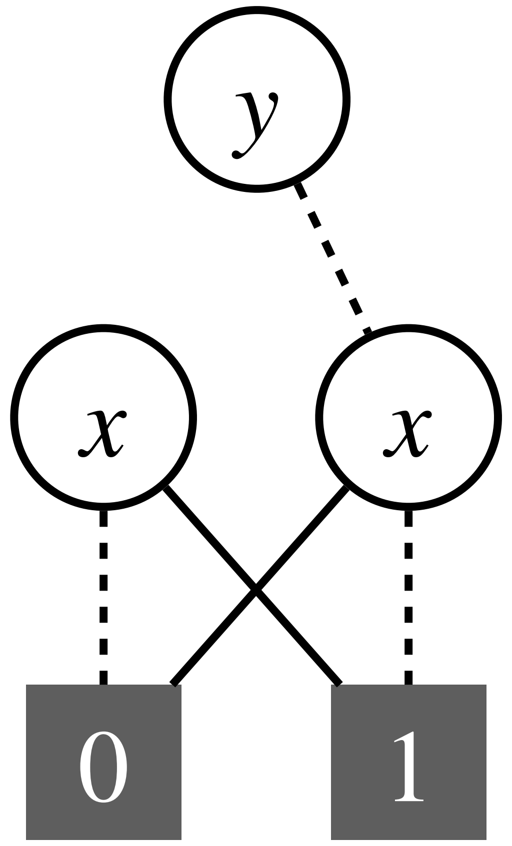

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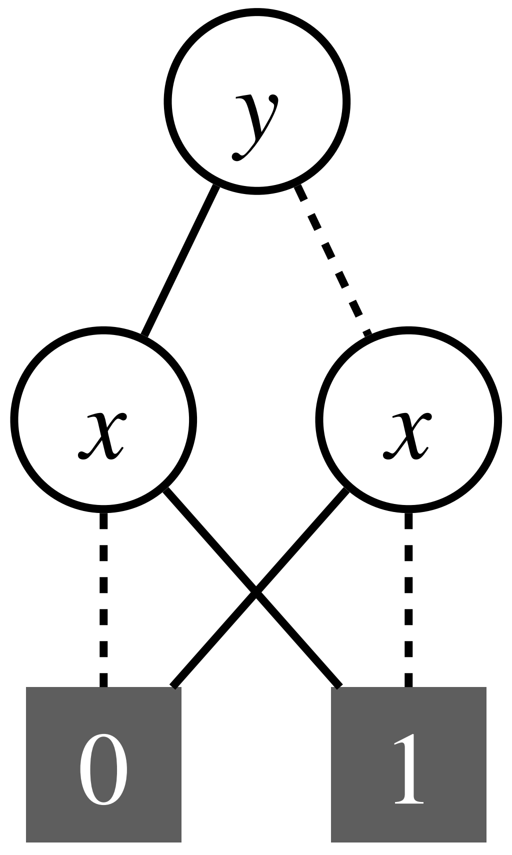

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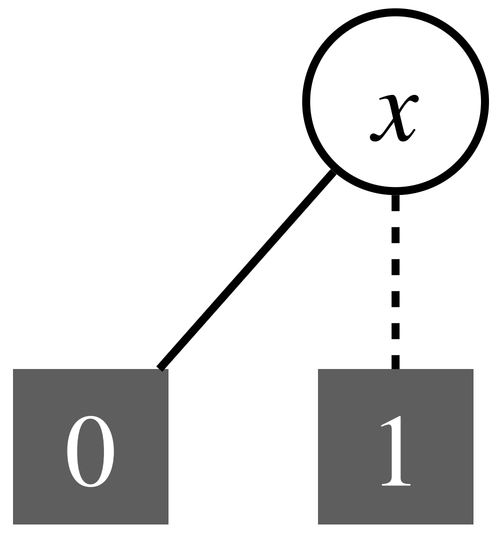

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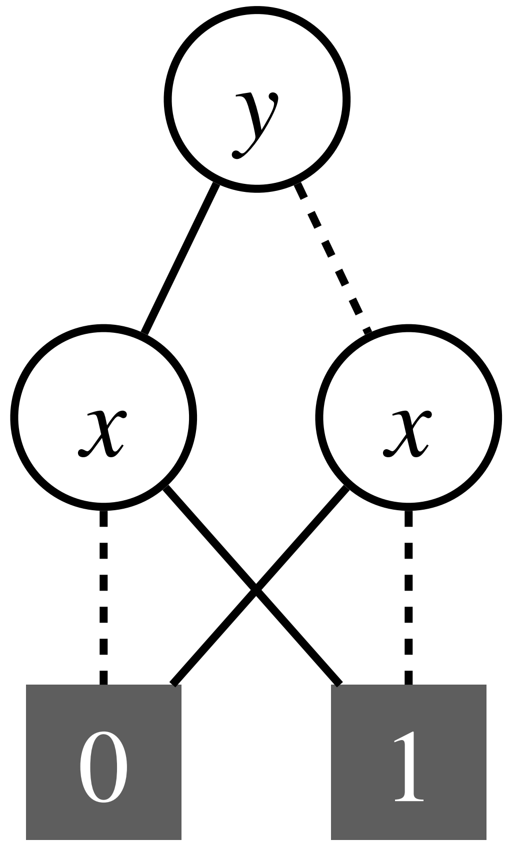

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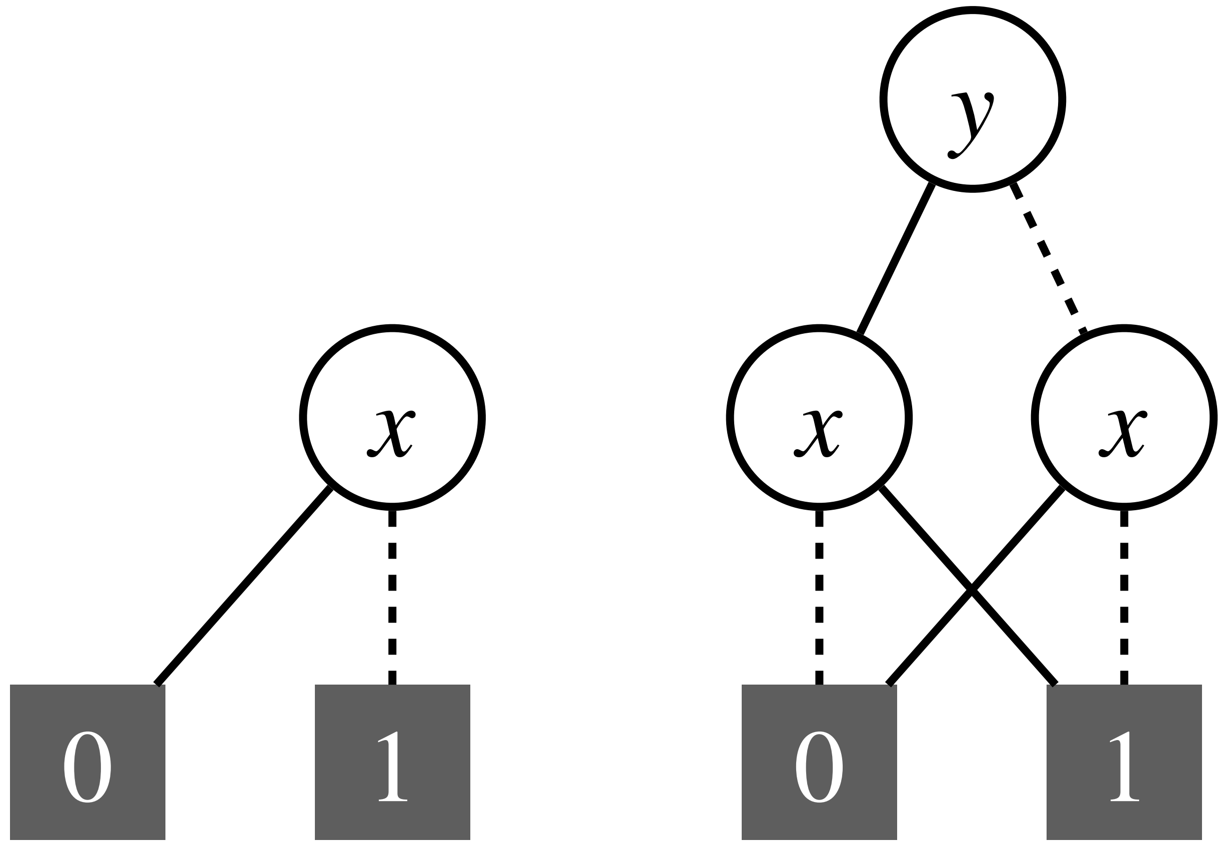

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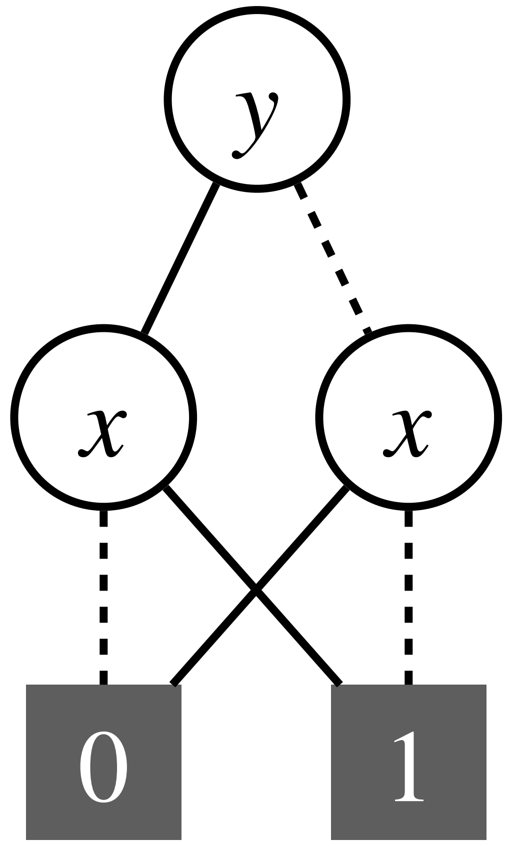

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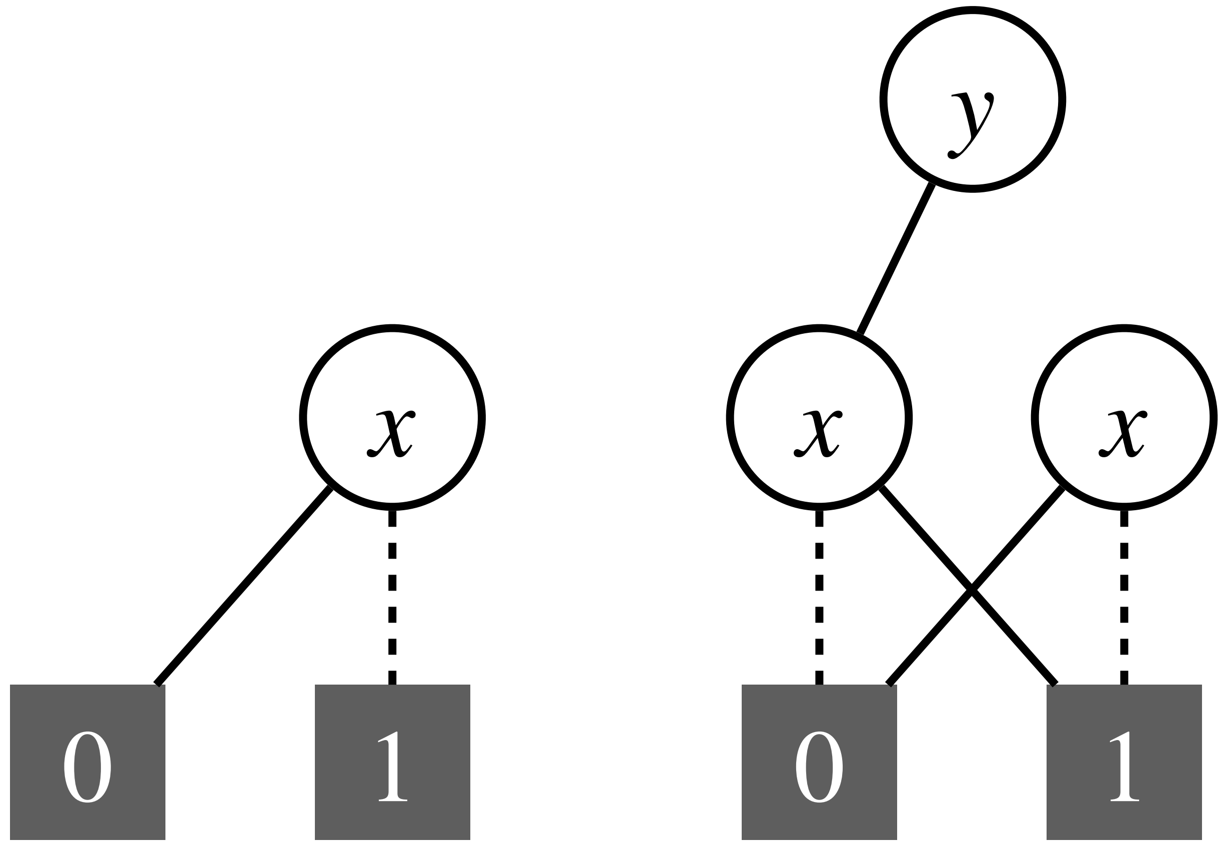

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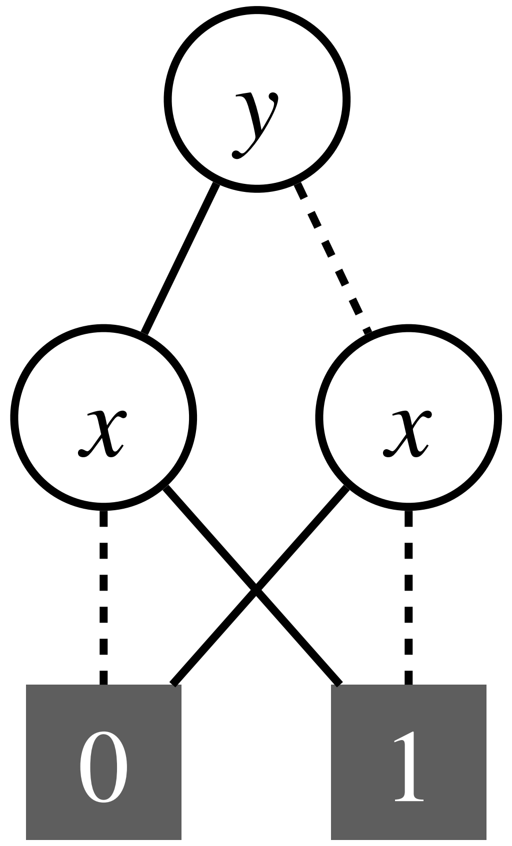

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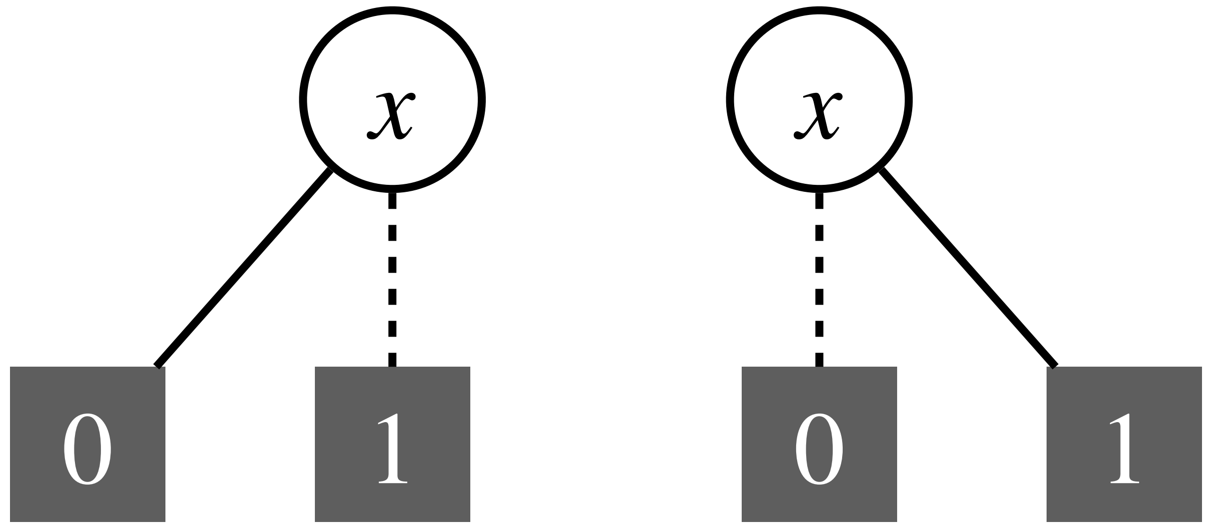

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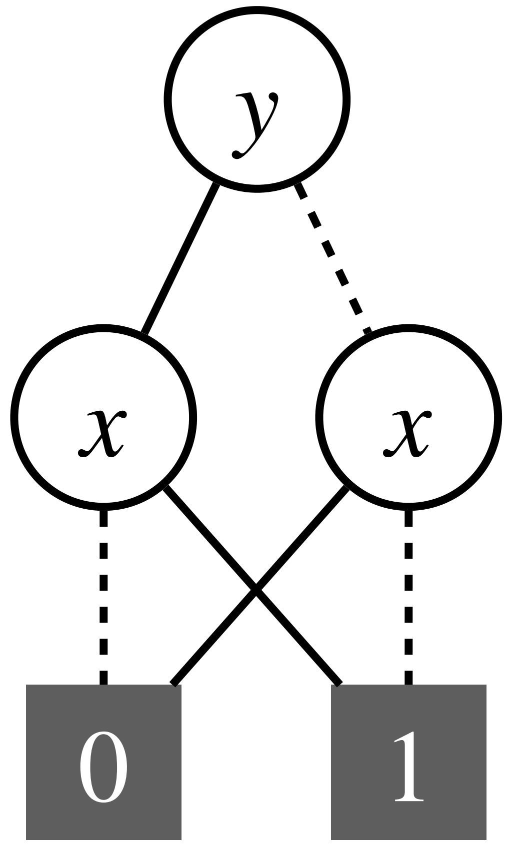

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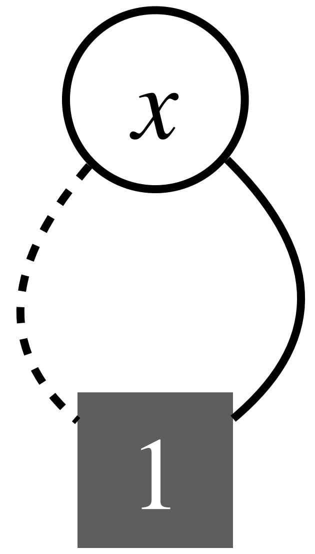

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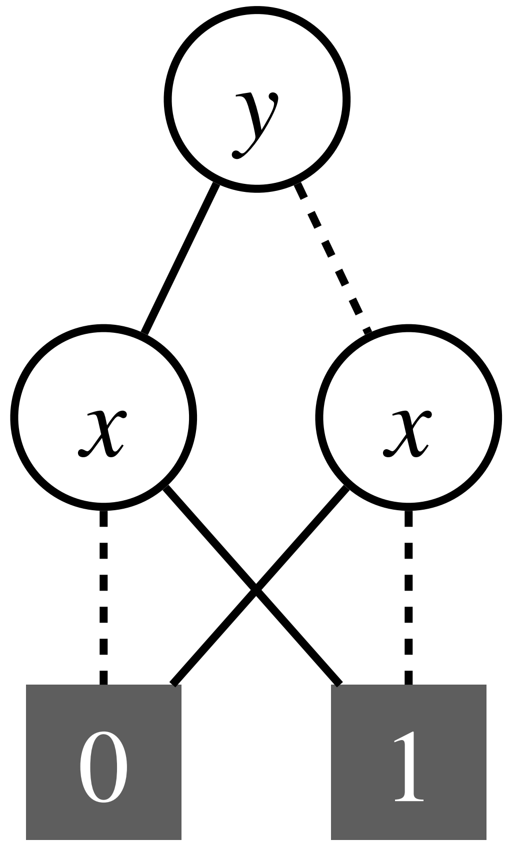

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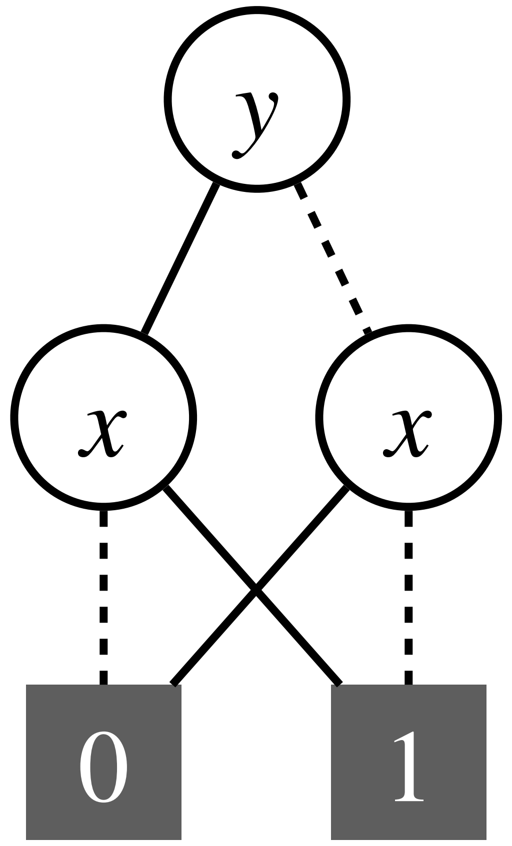

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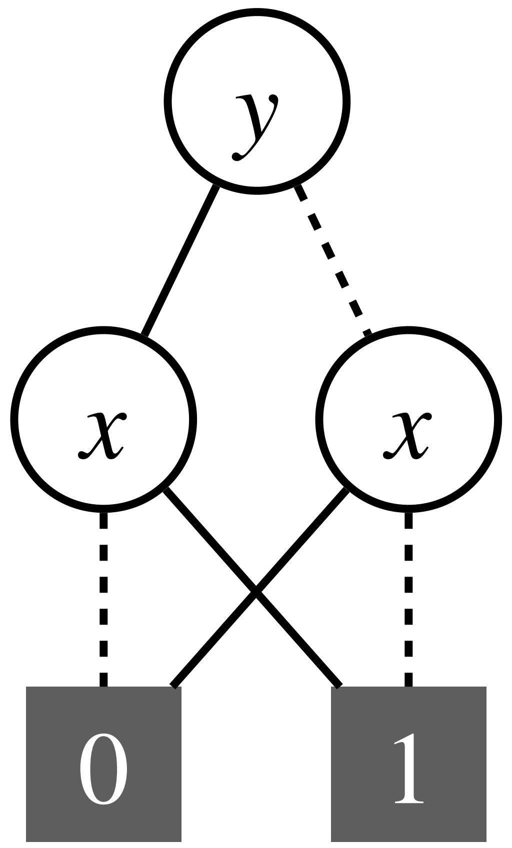

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$$Q_1 x_1 Q_2 x_2 \ldots Q_n x_n \,.\, \varphi$$

$$Q_n x_n \bigwedge_{x_n \in C_j} C_j$$

Contributions

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proof system for symbolic QBF reasoning

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short proofs bounded pathwidth and quantifier alternation

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separations from

Contributions

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- Long-Distance QU-Resolution

Contributions

proof system for symbolic QBF reasoning **“OBDD proofs”**

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- Long-Distance QU-Resolution
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Contributions

proof system for symbolic QBF reasoning **“OBDD proofs”**

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separations from

- Long-Distance QU-Resolution
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- Bounded-Depth Frege + ∀-Reduction

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proof system for symbolic QBF reasoning **“OBDD proofs”**

short proofs bounded pathwidth and quantifier alternation

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exponential lower bounds (even with a SAT oracle)

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“genuine”

OBDD Proofs

OBDD Proofs

$$Q_1 x_1 Q_2 x_2 \ldots Q_n x_n \,.\, C_1 \land \ldots \land C_m$$

OBDD Proofs

$$Q_1 x_1 Q_2 x_2 \ldots Q_n x_n \,.\, C_1 \wedge \ldots \wedge C_m$$

$Q_i \in \{\exists, \forall\}$

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B_1

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$B_1 \quad B_2$

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OBDDs with same variable ordering

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OBDDs with same variable ordering

Axiom

OBDD Proofs

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$B_i \equiv C_j$

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Projection (∃)

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OBDDs with same variable ordering

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Conjunction (∧)

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$B_i \equiv B_j \wedge B_k$

OBDD Proofs

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x innermost in B_j

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OBDDs with same variable ordering

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OBDDs with same variable ordering

OBDD($\wedge,\exists,\forall$)

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Entailment ($\models$)

$$\bigwedge_{j<i} B_j \models B_i$$

Small Pathwidth and Quantifier Alternation

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Theorem (Ferrara, Pan, Vardi)

A CNF formula φ with n variables and pathwidth k has an OBDD of size $O(n2^k)$.

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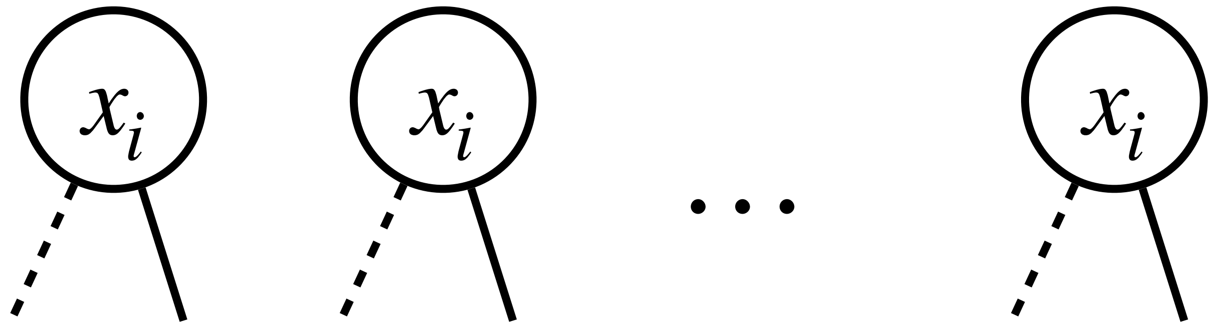

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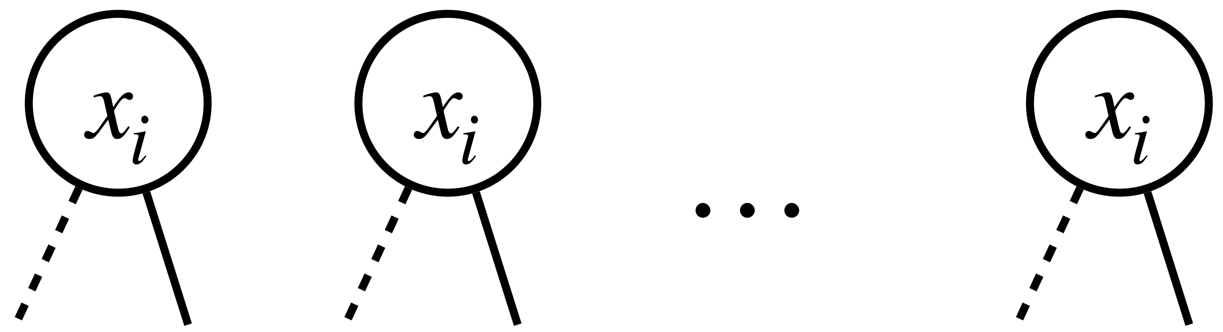
Lemma (Capelli, Mengel)

Let B be an OBDD of width w and $X \subseteq var(B)$. An OBDD for $\exists X \,.\, B$ of width 2^w can be computed in time $2^w |B|^{O(1)}$.



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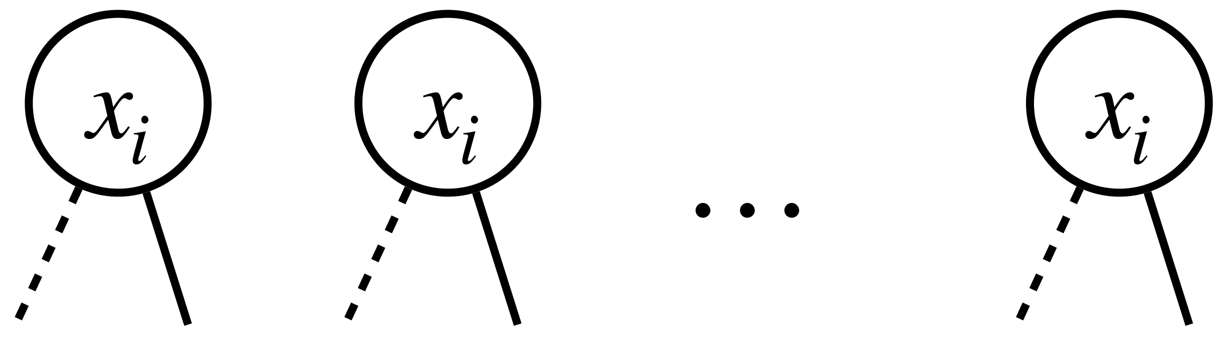

Corollary

Let $\mathscr{C}$ be a class of false QBFs of bounded pathwidth and quantifier alternation.
Then $\mathscr{C}$ has polynomial **OBDD**$(\wedge, \exists, \forall)$ proofs.

Separations

Separations

$$\Phi_n = \exists x_1 \ldots \exists x_n \forall u \exists t_2 \ldots \exists t_n \,.\, (t_2 \leftrightarrow (x_1 \oplus x_2)) \wedge \bigwedge_{i=3}^{n} (t_i \leftrightarrow (t_{i-1} \oplus x_i)) \wedge (u \leftrightarrow \neg t_n)$$

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Theorem (Beyersdorff, Bonacina, Chew, Pich)

The class $\{\Phi_n\}_{n \in \mathbb{N}}$ does not have polynomial proofs in AC^0-Frege+∀-Reduction.

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The class $\{\Phi_n\}_{n \in \mathbb{N}}$ has bounded pathwidth and quantifier alternation.

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Corollary

AC^0-Frege+∀-Reduction does not p-simulate **OBDD**($\wedge, \exists, \forall$).

Separations

Separations

Theorem

OBDD($\wedge, \exists, \forall$) is **not** p-simulated by

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Lower Bounds

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6. We define a QBF that has this function as a unique universal winning strategy.

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Summary & Conclusion

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Potential for symbolic quantifier elimination in a portfolio?

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genuine exponential lower bounds via synchronized communication complexity

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Can we use this connection for lower bounds against other proof systems?