SAT-Based Rigorous Explanations for Decision Lists

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July 7, 2021 | **SAT**

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eXplainable AI

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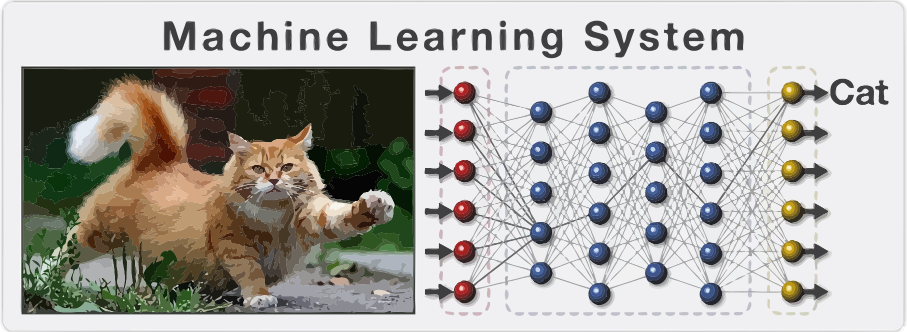
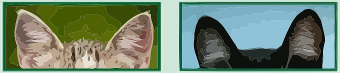


This is a cat.

Current Explanation

This is a cat:

- It has fur, whiskers, and claws.
- It has this feature:



XAI Explanation

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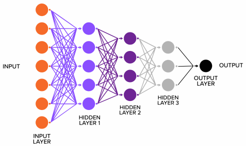
Why? Status quo...



	A parrot	Machine learning algorithm
Learns random phrases	✅	✅
Doesn’t understand s**t about what it learns	✅	✅
Occasionally speaks nonsense	✅	✅

©Internet

interpretable ML models

e.g. decision trees, lists, sets

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posthoc explanation of ML models "on the fly"

rule-based models

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⬇

“*transparent*” and easy to interpret

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come in handy in XAI

but...

Decision trees aren't interpretable

$$f(x_1, \ldots, x_n) = \bigvee_{i=1}^{n/2} x_{2i-1} \wedge x_{2i}, \text{ with } n = 4$$

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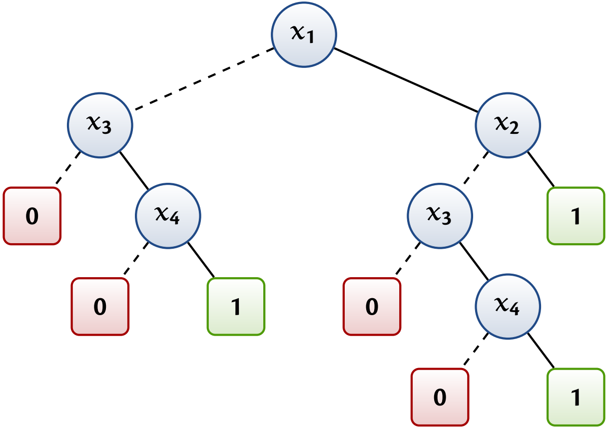

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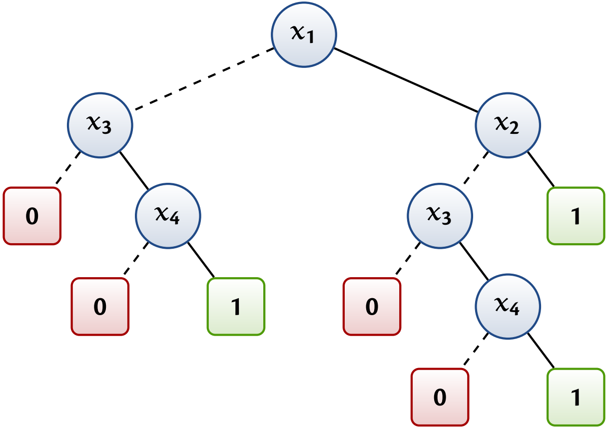

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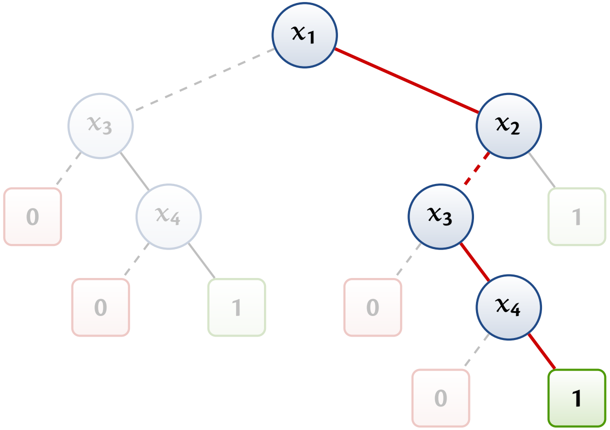

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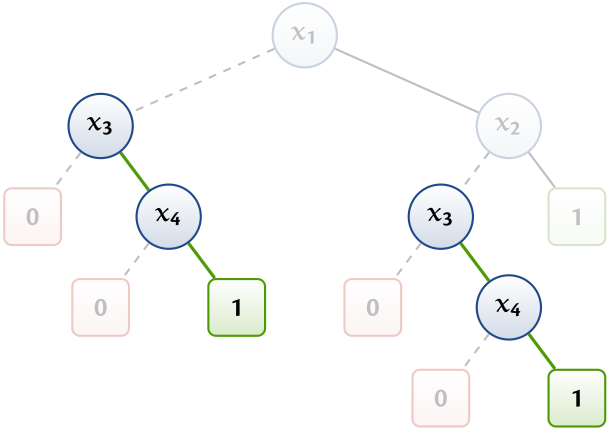

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actual explanation $x_3 = 1 \wedge x_4 = 1$ — 2 literals

DL explainability

classifier $\tau : \mathbb{F} \rightarrow \mathcal{K}$, **instance** $\mathbf{v}$ **s.t.** $\tau(\mathbf{v}) = c$

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contrastive explanation $\mathcal{Y}$

$$\exists(\mathbf{x} \in \mathbb{F}). \bigwedge\nolimits_{j \notin \mathcal{Y}} (x_j = v_j) \land (\tau(\mathbf{x}) \neq \mathbf{c})$$

DL example and duality

$$\mathbb{F} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}\}^{5} \qquad \mathcal{K} = \{\ominus, \oplus\}$$

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minimal hitting set duality!

Interpretability issue – just like with DTs

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actual AXp — $x_3 = 1 \wedge x_4 = 1$ — 2 literals

Are DLs hard to explain?

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see paper for details!

decision lists:

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in contrast to decision trees!

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Experimental results

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  - *targets* either AXps or CXps
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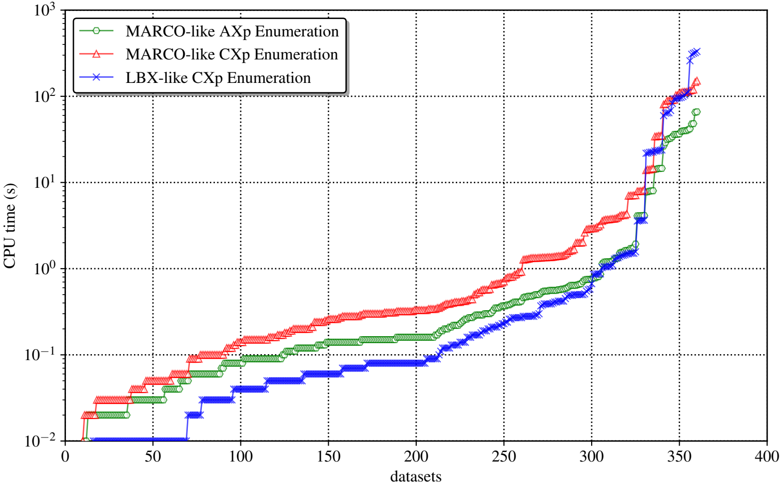
  - **minimum hitting sets** — RC2 MaxSAT
  - **XP reduction** — deletion-based linear search

Results – raw performance



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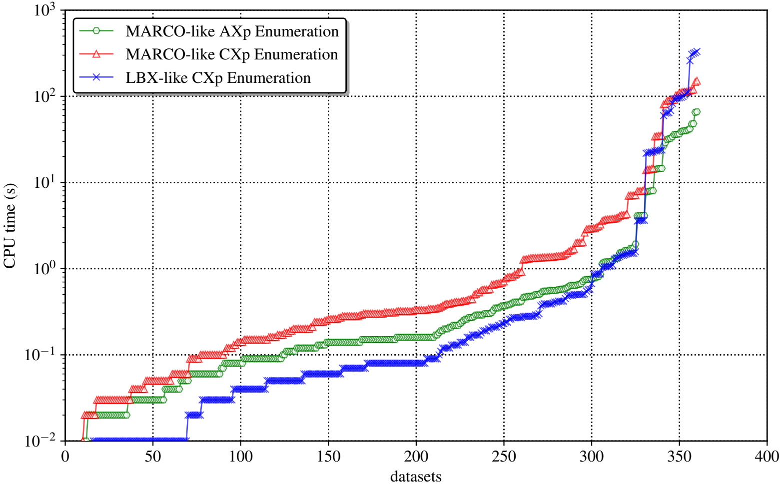

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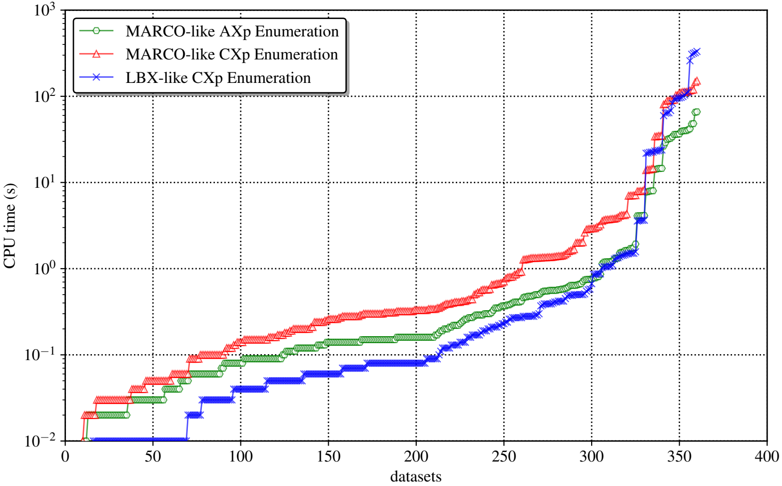

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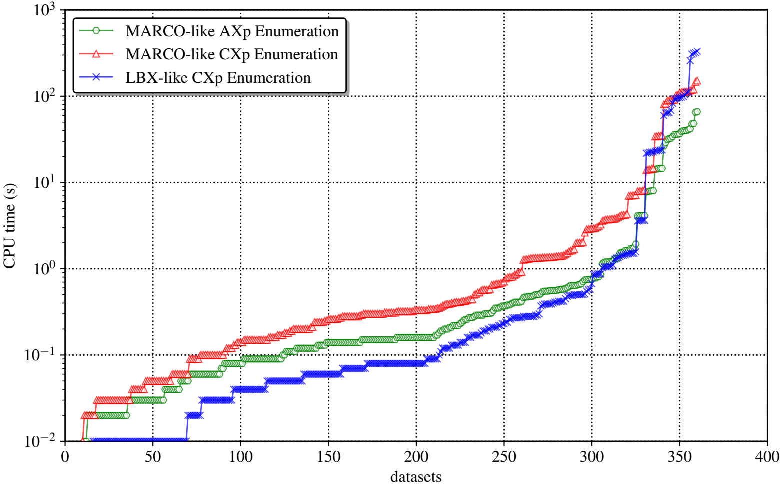

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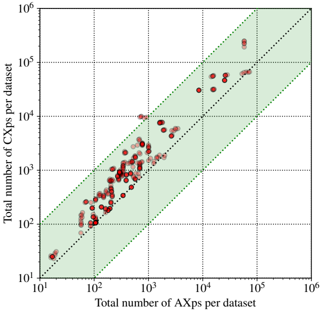
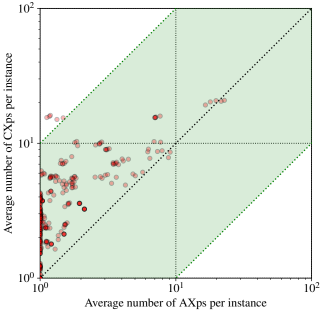
MARCO-like setup — targeting AXps may pay off

direct CXp enumeration is slower *(too many XPs?)*

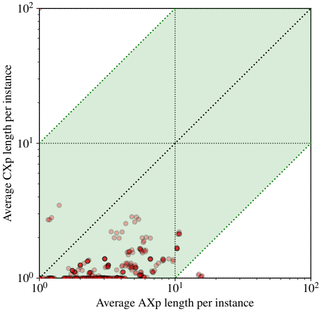
Results – AXps vs. CXps



(a) total number of AXps and CXps

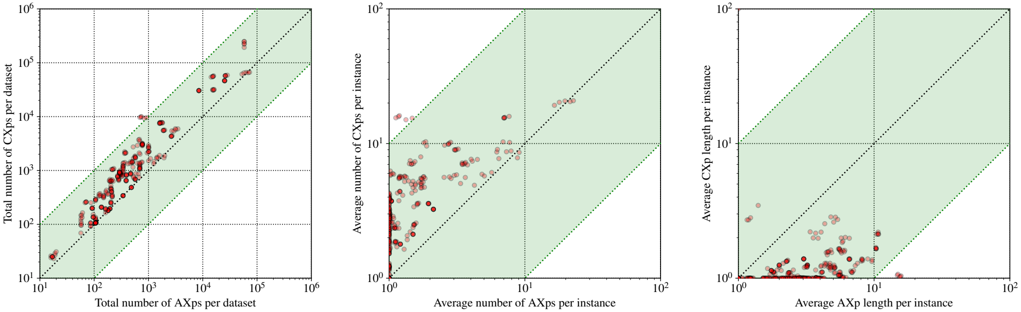


(b) avg. number of AXps and CXps



(c) avg. explanation size

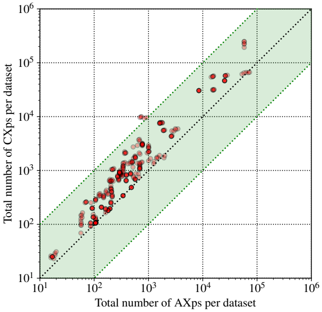
Results – AXps vs. CXps



(a) total number of AXps and CXps **(b)** avg. number of AXps and CXps **(c)** avg. explanation size

16–72838 AXps **vs.** **23–248825 CXps** *per dataset*

Results – AXps vs. CXps



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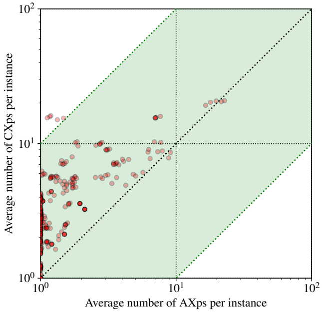

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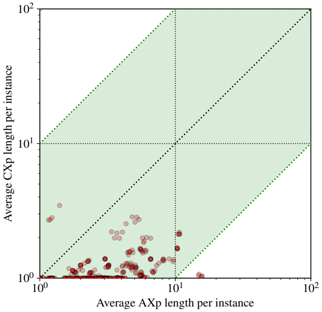

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1–22.7 AXps	**vs.**	**1–20.8 CXps**	*per instance*

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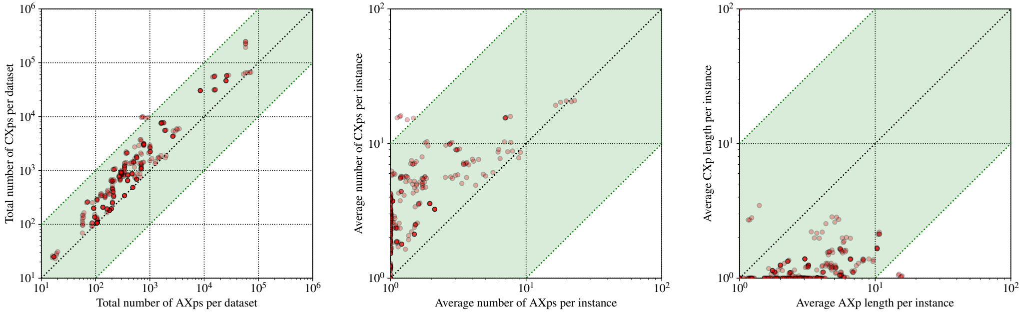

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16–72838 AXps	**vs.**	**23–248825 CXps**	*per dataset*
1–22.7 AXps	**vs.**	**1–20.8 CXps**	*per instance*
1–15.8 lits per AXp	**vs.**	**$\leq$2.8 lits per CXp**	

Summary

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  - **DLs may be uninterpretable**
    - just like decision trees!
  - **finding one explanation is not polytime, unless $\mathrm{P} = \mathrm{NP}$**
    - same for decision sets!
    - *and in contrast to decision trees!*

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Questions?