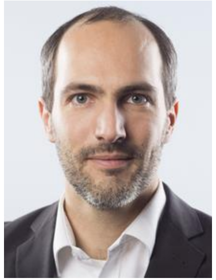
Solving Non-Uniform Planted and Filtered Random SAT Formulas Greedily



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Random k-SAT

- Draw m clauses independently at random:
  1. Draw k variables uniformly at random (without repetition)
  2. Negate each variable with probability $1/2$

$$\underbrace{X_3 \vee \overline{X_1} \vee \ldots \vee \overline{X_5}}_{k \text{ variables}}$$

Random k-SAT and solvability

$\sim 2^k \ln k / k$ — Solvable in polynomial time [Coja-Oghlan 2010]

$\sim 2^k \ln 2 - \frac{1 + \ln 2}{2} + o(k)$ — Satisfiability threshold [Ding, Sly and Sun 2015, …]

$\sim \ln n$ — Satisfiable instances solvable in polynomial time [Bulatov and Skvortsov 2015]

Ratio of clauses to variables m/n

Planted k-SAT

- Draw an assignment $\alpha \in \{0,1\}^n$ uniformly at random
- Draw m clauses independently at random:
  1. Draw k variables uniformly at random (without repetition)
  2. Draw one of the $2^k - 1$ negation patterns so that α satisfies the clause uniformly at random

$$\alpha = 11010$$

$$X_3 \vee \overline{X_1} \vee \overline{X_5}$$

$$X_3 \vee \overline{X_1} \vee X_5$$

Filtered SAT and Planted SAT

Planted k-SAT:

$\sim const$ — Solvable by spectral methods [Flaxman 2008]

$\sim \log n$ — Solvable by greedy methods [Bulatov and Skvortsov 2015]

Planted k-SAT ≈ Filtered k-SAT [Ben-Sasson, Bilu and Gutfreund 2002]

Ratio of clauses to variables m/n

Planted SAT: one hidden satisfying assignment

Filtered SAT: only satisfiable random k-SAT instances

Planted k-SAT ≠ Filtered k-SAT

Non-Uniform Random k-SAT [Ansótegui et al. 2009]

Draw m clauses independently at random:

1. Draw k variables according to $\overrightarrow{p^{(n)}}$ at random (without repetition)
2. Negate each variable with probability $1/2$

$\overrightarrow{p^{(n)}} = \left(p_1^{(n)}, p_2^{(n)}, \dots, p_n^{(n)}\right)$ - probability distribution over n Boolean variables

Our results

Greedy Algorithm [Koutsoupias and Papadimitriou 1992]

1. $\alpha \leftarrow$ assignment chosen uniformly at random;
2. **while** $\exists\, i \in [n]$: changing α_i increases the number of satisfied clauses **do**
   $\alpha_i \leftarrow 1 - \alpha_i$;
3. **return** α

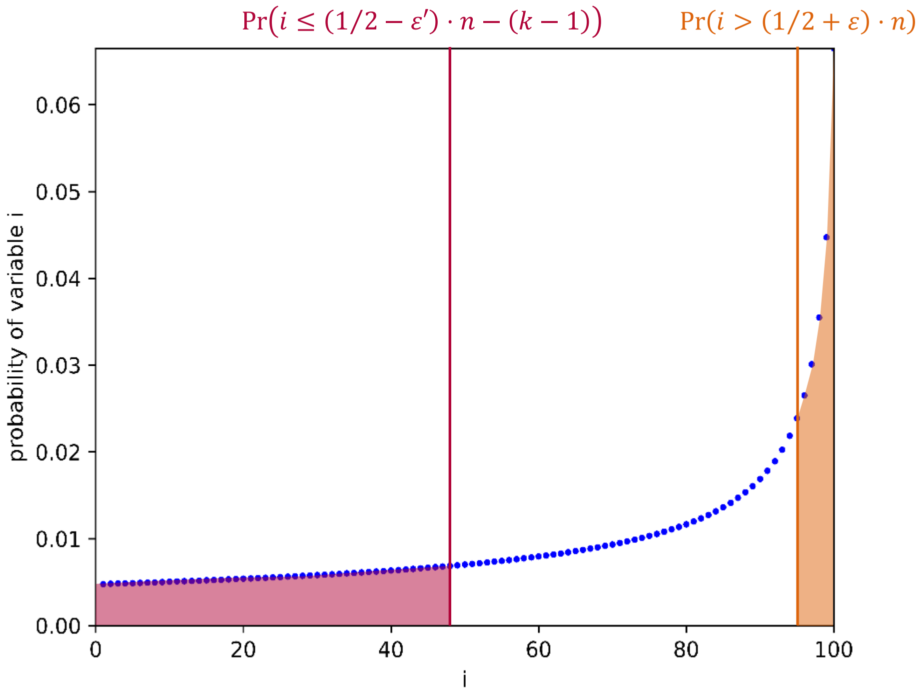
Theorem (simple version)

For non-uniform planted k-SAT with

- $k \geq 3$ constant
- „well-behaved" probability distribution
- sufficiently large m

the greedy algorithm succeeds with high probability.

Constraints on the probability distribution



$$p_1^{(n)} \le p_2^{(n)} \le \cdots \le p_n^{(n)}$$
$$\gamma_k(\varepsilon) \coloneqq \Pr(i \le (1/2 - \varepsilon) \cdot n - (k-1))$$

Well-behavedness:

There are constants $0 < \varepsilon' < \varepsilon < 1/2$ with

$\Pr\big(i \le (1/2 - \varepsilon') \cdot n - (k-1)\big)$

$> c + \Pr(i > (1/2 + \varepsilon) \cdot n)$

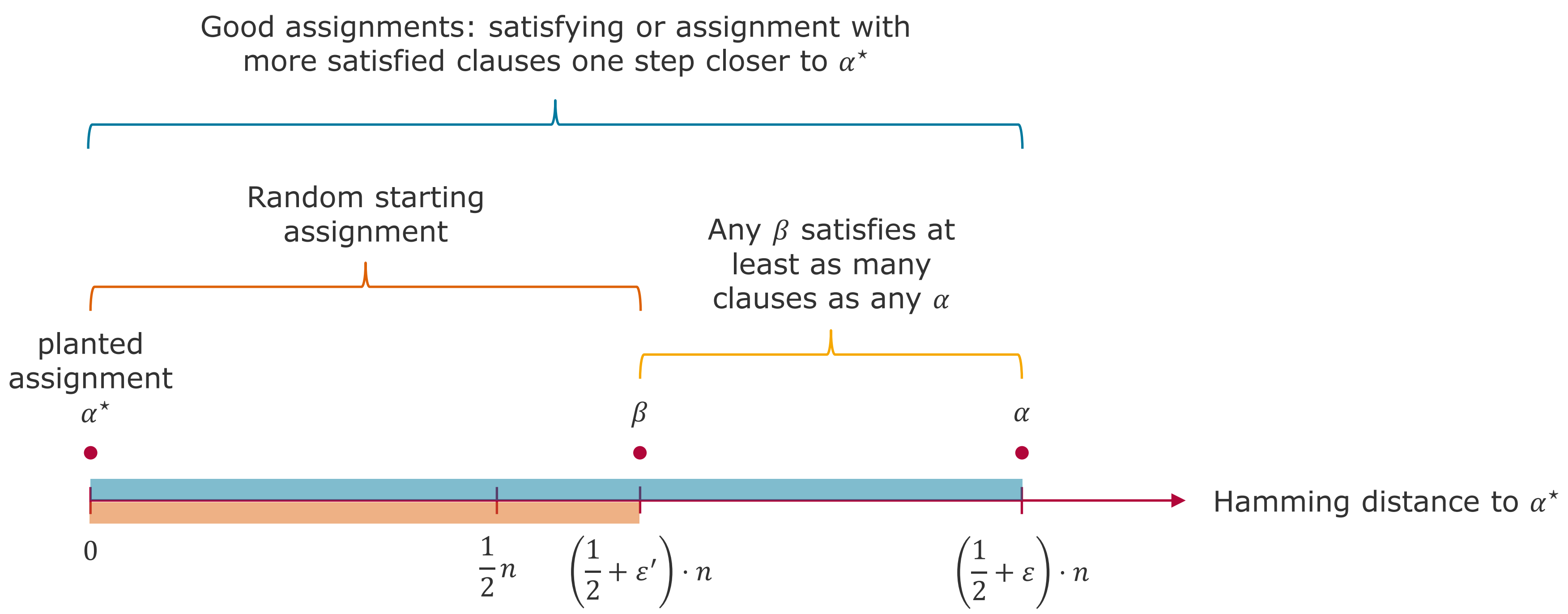
$$c = \Omega\left(\left(n \cdot p_1 \cdot \frac{\gamma_k(\varepsilon)^{3(k-1)}}{\ln n}\right)^{\frac{1}{2k}}\right)$$

$c \sim \ln^{-\frac{1}{2k}} n$ sufficient

Geedy succeeds w.h.p.:

$$m \ge \frac{C \ln n}{\gamma_k(\varepsilon)^{3(k-1)} p_1}$$

Proof Sketch



Planted vs Filtered

Geedy succeeds w.h.p. on Non-Uniform Planted k-SAT:

$$m \geq \frac{C \ln n}{\gamma_k(\varepsilon)^{3(k-1)} \cdot p_1}$$

Non-Uniform Planted k-SAT ≈ Non-Uniform Filtered k-SAT:

$$m \geq \frac{(1+c) \cdot \left(2^k - 1\right) \cdot \ln n}{p_1}$$

$\gamma_k(\varepsilon)$ constant

$p_1 \sim n^{-1}$

algorithm successful:

$m/n \sim \ln n$

- Uniform distribution
- Power law distribution
- Geometric distribution

Thank you!

References

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