Hardness and Optimality in QBF Proof Systems Modulo NP

Leroy Chew

TECHNISCHE UNIVERSITÄT WIEN

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- $\forall$ wins a game if the matrix becomes false.
- A QBF is false iff there exists a winning strategy for $\forall$.
- Strategy Extraction (from a refutation) allows one to extract, in polynomial-time, circuits $\{\sigma_1 \ldots \sigma_n\}$ that represent the winning strategy for $\forall$ variables $\{y_1, \ldots, y_n\}$

Extended QU-Resolution

$$\frac{}{C}\ \text{(Ax)} \qquad\qquad \frac{C \vee x \qquad D \vee \neg x}{C \vee D}\ \text{(Res)}$$

$$\frac{C \vee l}{C \vee 0}\ (\forall\text{-Red}) \qquad\qquad \frac{}{x \vee y \vee n, \bar{x} \vee \bar{n}, \bar{y} \vee \bar{n}}\ \text{(Ext.)}$$

$\forall$ literal l is quantified right of all variables in C.
New $\exists$ variable n is quantified right of x and y

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First Result: Equivalent to Extended Frege+$\forall$-Red

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- No more propositional lower bounds like Pigeonhole Principle.
- Analogous with the fact that SAT black boxes are used in QBF solvers.
- Technically, we are no longer working in the Cook-Reckhow definition of a proof system (unless $P = NP$)

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Two important things!

- QBF solvers frequently use SAT solvers as black boxes.
- You might not only want to know the truth value of QBF but the strategy (e.g. chess).

Towards Certification in QBF

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Goal: More rigorous and reliable QBF practical solving.

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How would this help?

Move towards a unified checking format which captures all QBF techniques

Extended QU-Res Normal Form

Split proof into two parts:

1st part: Purely Propositional Part

2nd part: Dual $\forall$-Red

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$$(0 \neq \sigma_{y_n}) \vee \bigvee_{i=1}^{n-1}(y_i \neq \sigma_{y_i}) \qquad\qquad (1 \neq \sigma_{y_n}) \vee \bigvee_{i=1}^{n-1}(y_i \neq \sigma_{y_i})$$

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Suppose S is a QBF refutation system and has polynomial time strategy extraction in circuits

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- In Merge Resolution each line (L_j) is a pair $(C_j, \{M_j^{y_i} \mid 1 \leq i \leq n\})$.

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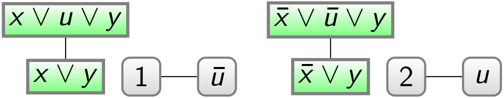

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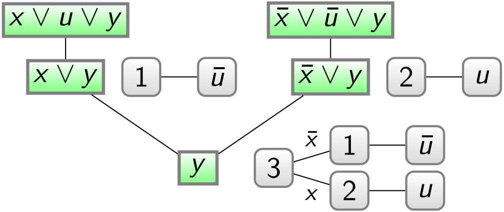

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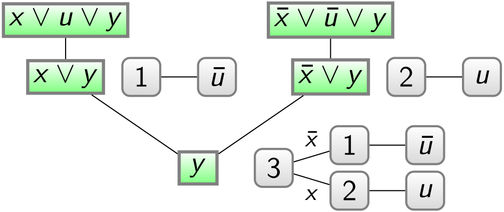

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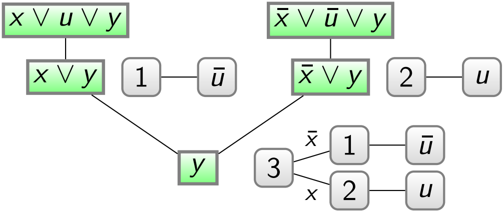

- Rephrase this as propositional logic $\bigwedge_{i=1}^{n}(y_i = M_j^{y_i}) \rightarrow C_j$.
- Easy to simulate Strat(M-Res) with Ext. Res,
  - Ext. variables represent nodes in the merge maps
  - Merge cases argued propositionally

Simulating Merge Resolution

1st part: Purely Propositional Part

$$
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What if Ext Res doesn't simulate $Strat(S)$?
Then Ext Res$+\|\mathrm{refl}(Strat(S))\|$ simulates $Strat(S)$.

Main Theorems

Theorem

For QBF Proof System S that has strategy extraction, Ext QU-Res $+\|\mathrm{refl}(\mathit{Strat}(S))\|$ simulates S.

Definition (Messner, Toran 98)

A proof system in language $\mathcal{L}$ is optimal if and only if it can simulate all other proof systems for $\mathcal{L}$.

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Ext QU-Res, when augmented with an NP oracle is optimal among all QBF proof systems with strategy extraction.

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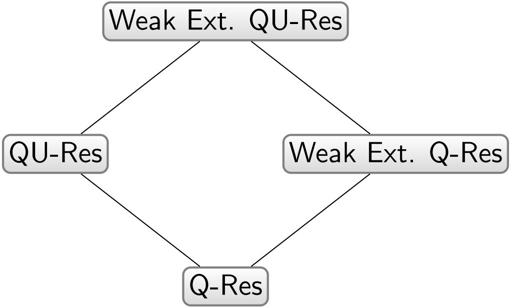
2nd part: Dual $\forall$-Red

Remove each disjunct inductively, as before

□

Hardness for Weaker Calculi



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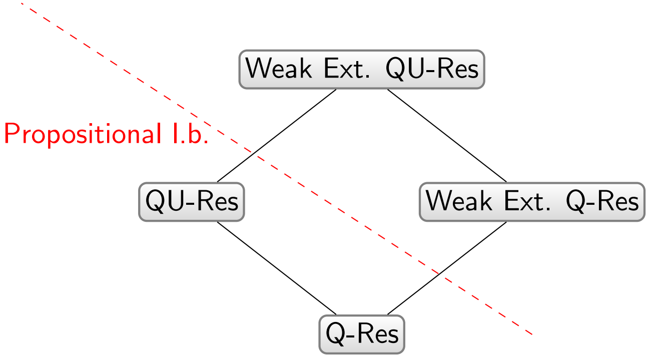

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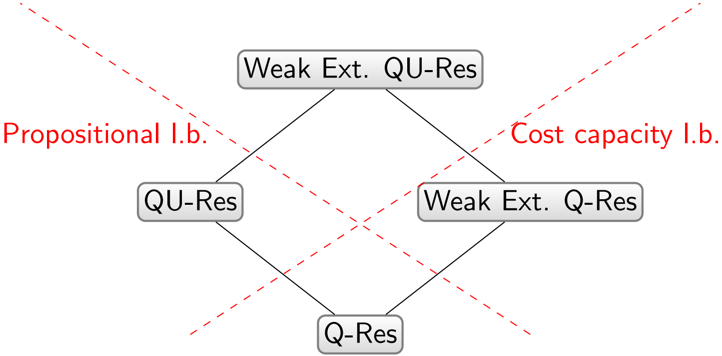

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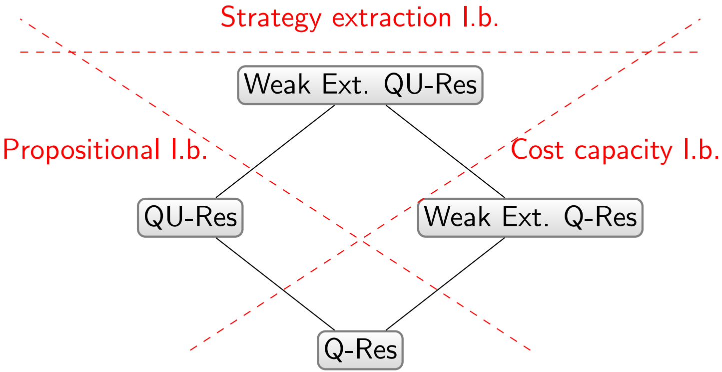
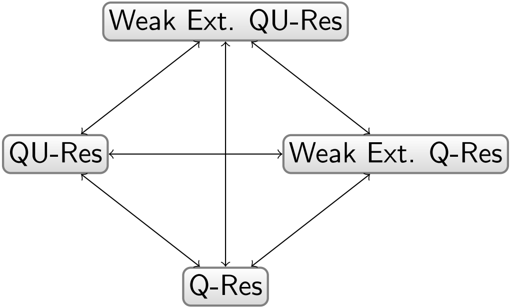


Collapse under NP Oracles

It can be shown that Q-Res can simulate a weak extended QU-Res proof



Simulation

We leave in the reduction steps but mimic all in-between inferences with NP oracles since the inference is just propositional.

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- W Ext QU-Res, W Ext Q-Res, QU-Res, Q-Res all are separated, but collapse with an NP oracle.