Davis and Putnam Meet Henkin:

Solving DQBF with Resolution

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\overbrace{\forall u_1 \exists x_1 \forall u_2 \exists x_2}^{\text{QBF}} &= \forall u_1 \forall u_2 \exists x_1(u_1) \exists x_2(u_1, u_2) \\
? &= \underbrace{\forall u_1 \forall u_2 \exists x_1(u_1) \exists x_2(u_2)}_{\text{DQBF}}
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- DQBF is NEXP-complete (vs QBF's PSPACE);
- we lift DP-resolution to DQBF.

Context

- There is a number of resolution-based proof systems for QBF, notably $\forall$Exp+Res [10] with its derivatives IR-calc and IRM-calc [4], and Q-Res [11] with its generalizations QU-Res [8], LDQ-Res [1, 7], and LDQU-Res [2];

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- Of these, only $\forall$Exp+Res and IR-calc lift to DQBF [5];
- Our algorithm builds on the idea of yet another proof system, M-Res [3], which itself has roots in LDQ-Res.

Propositional DP-resolution

$$(x \vee \overline{y}) \wedge (y \vee \overline{z}) \wedge (z \vee \overline{x}) \quad \wedge \quad (x \vee y \vee z) \wedge (\overline{x} \vee \overline{y} \vee \overline{z})$$

Propositional DP-resolution

$$(x \vee \overline{y}) \wedge (y \vee \overline{z}) \wedge (z \vee \overline{x}) \quad \wedge \quad (x \vee y \vee z) \wedge (\overline{x} \vee \overline{y} \vee \overline{z})$$

Eliminate x:

Propositional DP-resolution

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Eliminate x: $(\overline{y} \vee z)$

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Eliminate x: $(\overline{y} \vee z) \wedge (\overline{y} \vee \overline{z})$

Propositional DP-resolution

$$(x \lor \overline{y}) \land (y \lor \overline{z}) \land (z \lor \overline{x}) \quad \land \quad (x \lor y \lor z) \land (\overline{x} \lor \overline{y} \lor \overline{z})$$

Eliminate x: $(\overline{y} \lor z) \land (\overline{y} \lor \overline{z})$

Propositional DP-resolution

$$(x \lor \overline{y}) \land (y \lor \overline{z}) \land (z \lor \overline{x}) \quad \land \quad (x \lor y \lor z) \land (\overline{x} \lor \overline{y} \lor \overline{z})$$

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Eliminate y:

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$$(x \lor \overline{y}) \land (y \lor \overline{z}) \land (z \lor \overline{x}) \quad \land \quad (x \lor y \lor z) \land (\overline{x} \lor \overline{y} \lor \overline{z})$$

Eliminate x: $(\overline{y} \lor z) \land (\overline{y} \lor \overline{z}) \land (y \lor z) \land (y \lor z \lor \overline{y} \lor \overline{z})$
Eliminate y: (z)

Propositional DP-resolution

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Eliminate x: $(\overline{y} \vee z) \wedge (\overline{y} \vee \overline{z}) \wedge (y \vee z) \wedge (y \vee z \vee \overline{y} \vee \overline{z})$

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Eliminate y: $(z) \wedge (\overline{z})$

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Eliminate x: $(\overline{y} \vee z) \wedge (\overline{y} \vee \overline{z}) \wedge (y \vee z) \wedge (y \vee z \vee \overline{y} \vee \overline{z})$
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$$(x \lor \overline{y}) \land (y \lor \overline{z}) \land (z \lor \overline{x}) \quad \land \quad (x \lor y \lor z) \land (\overline{x} \lor \overline{y} \lor \overline{z})$$

Eliminate x: $(\overline{y} \lor z) \land (\overline{y} \lor \overline{z}) \land (y \lor z) \land (y \lor z \lor \overline{y} \lor \overline{z})$

Eliminate y: $(z) \land (\overline{z}) \land (z \lor \overline{z})$

Eliminate z: $\bot$

H-form DQBF

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

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$$\neg\Big[\quad \forall x_1 \forall x_2 \exists u_1(x_1) \exists u_2(x_2) \quad (x_1 \wedge u_2) \vee (\overline{x_1} \wedge \overline{u_2}) \vee (x_2 \wedge u_1) \vee (\overline{x_2} \wedge \overline{u_1}) \quad \Big]$$

$$\Updownarrow$$

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- This formula is false because substituting the dependency-respecting functions $u_1 := x_1$ and $u_2 := \overline{x_2}$ (a countermodel) results in the unsatisfiable propositional formula

$$(x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2) \wedge (x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2}).$$

Algorithm Summary

$$\Psi \longmapsto \mathrm{DP}(\Psi, 0) \longmapsto \mathrm{DP}(\Psi, 1) \longmapsto \cdots \longmapsto \mathrm{DP}(\Psi, n) = \mathrm{DP}(\Psi)$$

- First, Ψ has to be transformed into the set of clause-strategy pairs $\mathrm{DP}(\Psi, 0)$;

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- in each step we have to resolve all pairs of clauses, adding the pivot if necessary (w-resolution);
- a resolution step is only omitted if it results in an existential tautology, or if strategies mismatch;
- clauses that did contain the pivot are discarded, others are carried forward;
- the resulting set $\mathrm{DP}(\Psi)$ consists of clause-strategy pairs with empty existential parts and countermodel strategies if the formula is false; otherwise it is empty.

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\begin{aligned}\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad & (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),\\ & (x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})\end{aligned}$$

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Eliminate x_1:

H-form DQBF DP-resolution

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$$\downarrow$$

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$$\downarrow$$

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Eliminate x_1: $(x_2;$

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$$(\overline{x_1} \vee x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\longleftarrow\!\!\dashv} u_1 \overset{x_1}{\mapsto} 0,$

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$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

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$$(\overline{x_1} \vee x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$,

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$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

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$$\downarrow$$

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$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_1} \vee \overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$,

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$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_1} \vee \overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0),\ (\overline{x_2};$

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_1} \vee \overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 1,$

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_1} \vee \overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\hookrightarrow} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\hookrightarrow} 1, u_2 \mapsto 0)$,

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_1 \vee x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \xleftarrow{\overline{x_1}} u_1 \xmapsto{x_1} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \xleftarrow{\overline{x_1}} u_1 \xmapsto{x_1} 1, u_2 \mapsto 0)$,

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_1 \vee x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,

$(x_2;$

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_1 \vee x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\leftarrow\!\!\dashv} u_1 \overset{x_1}{\mapsto} *,$

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \lor u_2) \land (\overline{x_1} \lor \overline{u_2}) \land (x_2 \lor u_1) \land (\overline{x_2} \lor \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_1 \lor x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0),\ (\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0),$
$(x_2;\ 0 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1),$

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$,

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (x_1 \vee \overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$,

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (x_1 \vee \overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};$

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (x_1 \vee \overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0),\ (\overline{x_2};\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0),$
$(x_2;\ 0 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1),\ (\overline{x_2};\ 1 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} *,$

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (x_1 \vee \overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$.

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$.

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0),\ (\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0),$
$(x_2;\ 0 \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1),\ (\overline{x_2};\ 1 \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1).$

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0),\ (\overline{x_2};\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0),$
$(x_2;\ 0 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1),\ (\overline{x_2};\ 1 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1).$

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$.

Eliminate x_2:

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$.

Eliminate x_2:

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0),\ (\overline{x_2};\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0),$
$(x_2;\ 0 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1),\ (\overline{x_2};\ 1 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1).$

Eliminate x_2:

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\longleftarrow} u_1 \overset{x_1}{\longmapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\longleftarrow} u_1 \overset{x_1}{\longmapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\longleftarrow} u_1 \overset{x_1}{\longmapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\longleftarrow} u_1 \overset{x_1}{\longmapsto} *, u_2 \mapsto 1)$.

Eliminate x_2:

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$.

Eliminate x_2:

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$.

Eliminate x_2:

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
$$(x_2;\ \{u_1 \mapsto 0, u_2 \mapsto *\}), (\overline{x_2};\ \{u_1 \mapsto 1, u_2 \mapsto *\})$$

Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0),\ (\overline{x_2};\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0),$
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$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$.

Eliminate x_2:

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \lor u_2) \land (\overline{x_1} \lor \overline{u_2}) \land (x_2 \lor u_1) \land (\overline{x_2} \lor \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
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H-form DQBF DP-resolution

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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$.

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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$.

Eliminate x_2: $(\quad;\ 1 \overset{\overline{x_1}}{\hookleftarrow} u_1 \overset{x_1}{\mapsto} 0,$

H-form DQBF DP-resolution

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
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Eliminate x_2: $(\quad ;\ 1 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 0, 0 \overset{\overline{x_2}}{\mapsfrom} u_2 \overset{x_2}{\mapsto} 1)$,

H-form DQBF DP-resolution

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$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\shortmid} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0),\ (\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0),$
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Eliminate x_2: $(\quad;\ 1 \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, 0 \overset{\overline{x_2}}{\leftarrow\!\dashv} u_2 \overset{x_2}{\mapsto} 1),$

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$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\mid} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\mid} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
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$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \lor u_2) \land (\overline{x_1} \lor \overline{u_2}) \land (x_2 \lor u_1) \land (\overline{x_2} \lor \overline{u_1})$$

$$\downarrow$$

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1;\ \{u_1 \mapsto *, u_2 \mapsto 0\}), (\overline{x_1};\ \{u_1 \mapsto *, u_2 \mapsto 1\}),$$
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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
$(x_2;\ 0 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$, $(\overline{x_2};\ 1 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} *, u_2 \mapsto 1)$.

Eliminate x_2: $(\quad;\ 1 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 0, 0 \overset{\overline{x_2}}{\mapsfrom} u_2 \overset{x_2}{\mapsto} 1)$, $(\quad;\ 0 \overset{\overline{x_1}}{\mapsfrom} u_1 \overset{x_1}{\mapsto} 1,$

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$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

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Eliminate x_1: $(x_2;\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 0, u_2 \mapsto 0)$, $(\overline{x_2};\ * \overset{\overline{x_1}}{\leftarrow\!\dashv} u_1 \overset{x_1}{\mapsto} 1, u_2 \mapsto 0)$,
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Eliminate x_2: $(\ ;\ 1 \xleftarrow{\overline{x_1}}\!\dashv u_1 \xmapsto{x_1} 0, 0 \xleftarrow{\overline{x_2}}\!\dashv u_2 \xmapsto{x_2} 1),\ (\ ;\ 0 \xleftarrow{\overline{x_1}}\!\dashv u_1 \xmapsto{x_1} 1, 1 \xleftarrow{\overline{x_2}}\!\dashv u_2 \xmapsto{x_2} 0) = \mathrm{DP}(\Psi)$

Algorithm Summary

$$\Psi \longmapsto \mathrm{DP}(\Psi, 0) \longmapsto \mathrm{DP}(\Psi, 1) \longmapsto \cdots \longmapsto \mathrm{DP}(\Psi, n) = \mathrm{DP}(\Psi)$$

- First, Ψ has to be transformed into the set of clause-strategy pairs $\mathrm{DP}(\Psi, 0)$;
- next, we eliminate existential variables one by one in any order;
- in each step we have to resolve all pairs of clauses, adding the pivot if necessary (w-resolution);
- a resolution step is only omitted if it results in an existential tautology, or if strategies mismatch;
- clauses that did contain the pivot are discarded, others are carried forward;
- the resulting set $\mathrm{DP}(\Psi)$ consists of clause-strategy pairs with empty existential parts and countermodel strategies if the formula is false; otherwise it is empty.

Correctness of the Algorithm

Theorem

$\mathrm{DP}(\Psi)$ *is non-empty if, and only if,* Ψ *is false.*

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DP(Ψ) *is non-empty if, and only if,* Ψ *is false.*

Proof.

$\Rightarrow$: prove that in any clause-strategy pair (C, h), h is a partial countermodel against $\Psi[\overline{C}]$;
$\Leftarrow$: prove that any countermodel h is represented in each DP(Ψ, i). □

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Theorem

$\mathrm{DP}(\Psi) \subseteq \mu(\Psi)$: *every minimal countermodel is generated.*

Complexity

- Side-result: H-form DQBF is NEXP-complete (via generalized Tseitin translation);

Complexity

- Side-result: H-form DQBF is NEXP-complete (via generalized Tseitin translation);
- worst-case running time $2^{2^{\mathrm{poly}(n)}} = \overbrace{\left(\left(|\Psi|^2\right)^{2^{\cdot^{\cdot^{\cdot}}}}\right)^2}^{n}$;

Complexity

- Side-result: H-form DQBF is NEXP-complete (via generalized Tseitin translation);
- worst-case running time $2^{2^{\text{poly}(n)}} = \left(\overbrace{\left(|\Psi|^2\right)^{2^{\cdot^{\cdot^{\cdot}}}}}^{n}\right)^2$;
- strategies can be represented by ordered binary decision diagrams (OBDDs), with polytime operations.

Remark

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \lor u_2) \land (\overline{x_1} \lor \overline{u_2}) \land (x_2 \lor u_1) \land (\overline{x_2} \lor \overline{u_1})$$

- Any DQBF with ≤ 3 variables is a QBF;

Remark

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

- Any DQBF with ≤ 3 variables is a QBF;
- the two parts of the formula share no variables;

Remark

$$\exists x_1 \exists x_2 \forall u_1(x_1) \forall u_2(x_2) \quad (x_1 \vee u_2) \wedge (\overline{x_1} \vee \overline{u_2}) \wedge (x_2 \vee u_1) \wedge (\overline{x_2} \vee \overline{u_1})$$

- Any DQBF with ≤ 3 variables is a QBF;
- the two parts of the formula share no variables;
- the formula is somehow still interesting...

Open Questions

- Reduce the number of w-resolution steps?
- Use as a preprocessing?

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