

# On quantale enriched monoids

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Highlighting the analogy between the triangle inequality for metric spaces and the categorical composition law, in 1973, Lawvere argued that “fundamental structures [in mathematics] are themselves categories” and thus, the latter should not be treated as a kind of “third level of abstraction”. Accordingly, he developed a framework in which a (generalized) metric space may be seen as a category enriched in the real half-line extended with the infinity value. More generally, for a quantale  $\mathcal{V}$ , a  $\mathcal{V}$ -category may be defined as a set equipped with a binary  $\mathcal{V}$ -valued relation satisfying reflexivity and transitivity axioms. For different choices of the quantale  $\mathcal{V}$ , we then obtain different mathematical structures, including preordered sets, the already mentioned Lawvere’s metric spaces, and probabilistic metric spaces.

In this talk we will consider monoids equipped with a compatible structure of  $\mathcal{V}$ -category, to which we call  $\mathcal{V}$ -monoids, and discuss some of their properties. In particular, we will investigate the possible quantale enrichments on semidirect products of  $\mathcal{V}$ -monoids as well as their connections to split extensions.